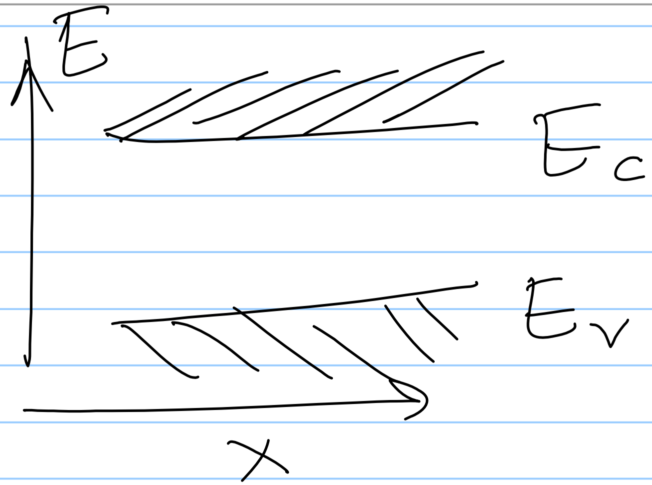
Energy versus Momentum



$$p \leftarrow F(x)$$

$p \leftarrow$ momentum operator

$$p \equiv \frac{\hbar}{i} \nabla = \frac{\hbar}{i}\left(\frac{\partial}{\partial x} + \frac{\partial}{\partial y} + \frac{\partial}{\partial z}\right)$$

$$\psi = \psi_0 \, e^{i(\omega t + kx)}$$

$\omega \rightarrow$ freq.

$k \rightarrow$ wave number

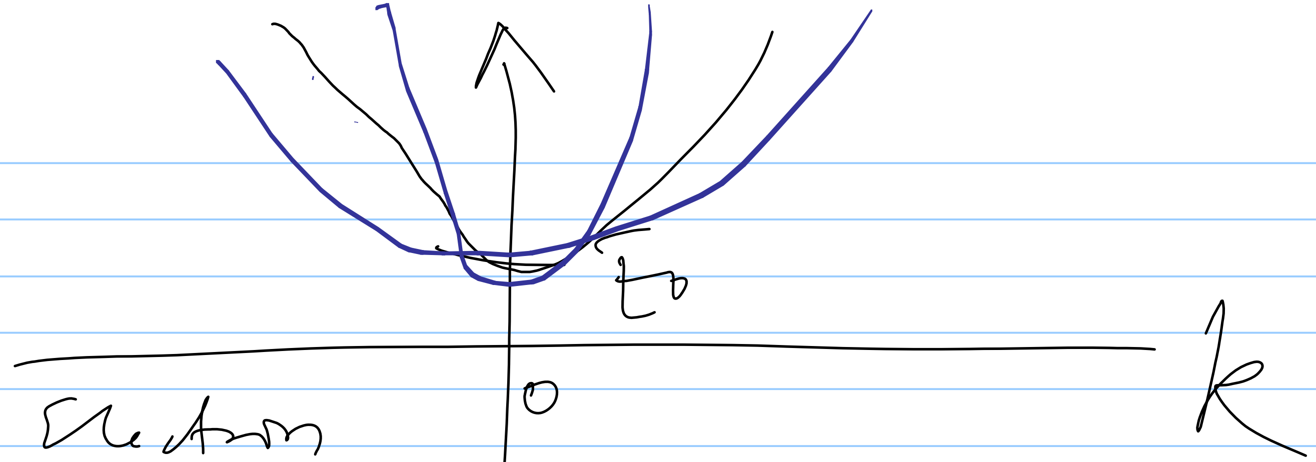
Wave fn: $\psi = \psi_0 e^{i(\omega t \mp kx)}$ ← free electron

$$p\psi = \frac{\hbar}{i}\frac{d}{dx}\psi = \frac{\hbar}{i} ik\psi = \hbar k\psi$$

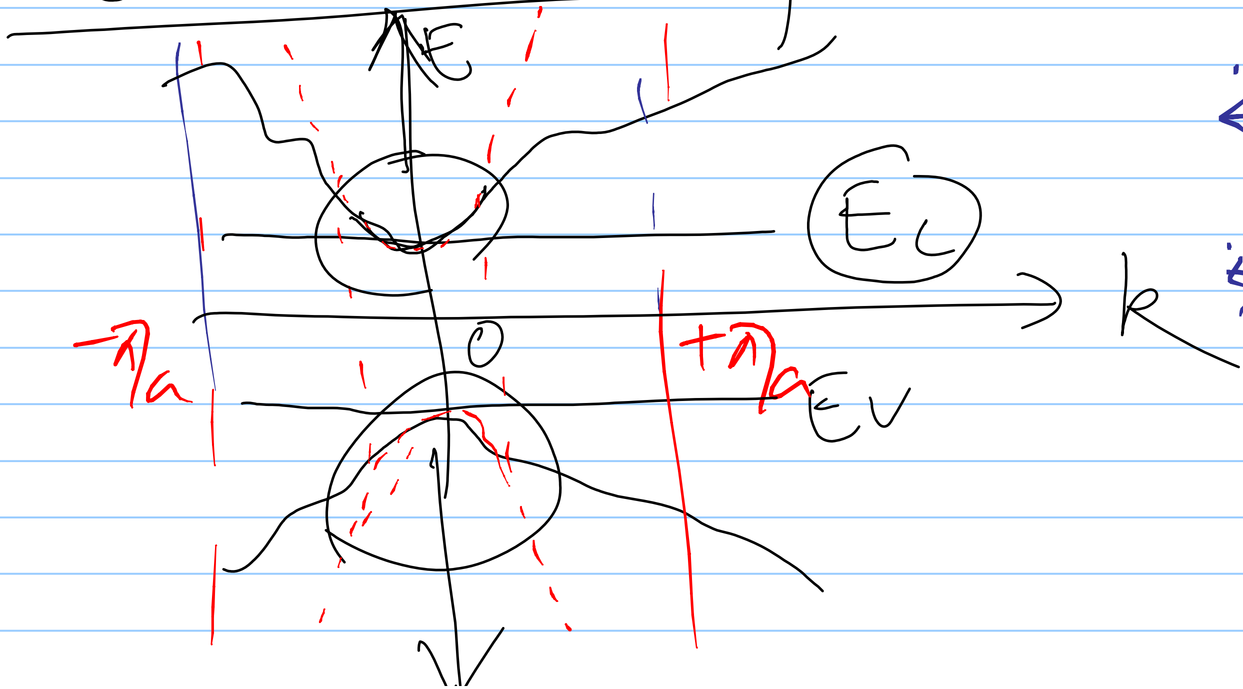
$$p = \hbar k$$

$$\text{Total Energy} = E_0 + \frac{1}{2}\frac{p^2}{m} = E_0 + \frac{\hbar^2 k^2}{2m}$$

$$\boxed{E = E_0 + \frac{\hbar^2 k^2}{2m}}$$



Crystal Electron



$\cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \longrightarrow x$

$\overleftrightarrow{a} \rightarrow$ lattice const

$\cdot \; \cdot \; \cdot \; \cdot \; \cdot \; \cdot \longrightarrow k$

$\overleftrightarrow{2\pi/a}$

$t \rightarrow T$

$f \rightarrow 2\pi/T$

$$E = E_0 + \frac{\hbar^2 k^2}{2m}$$

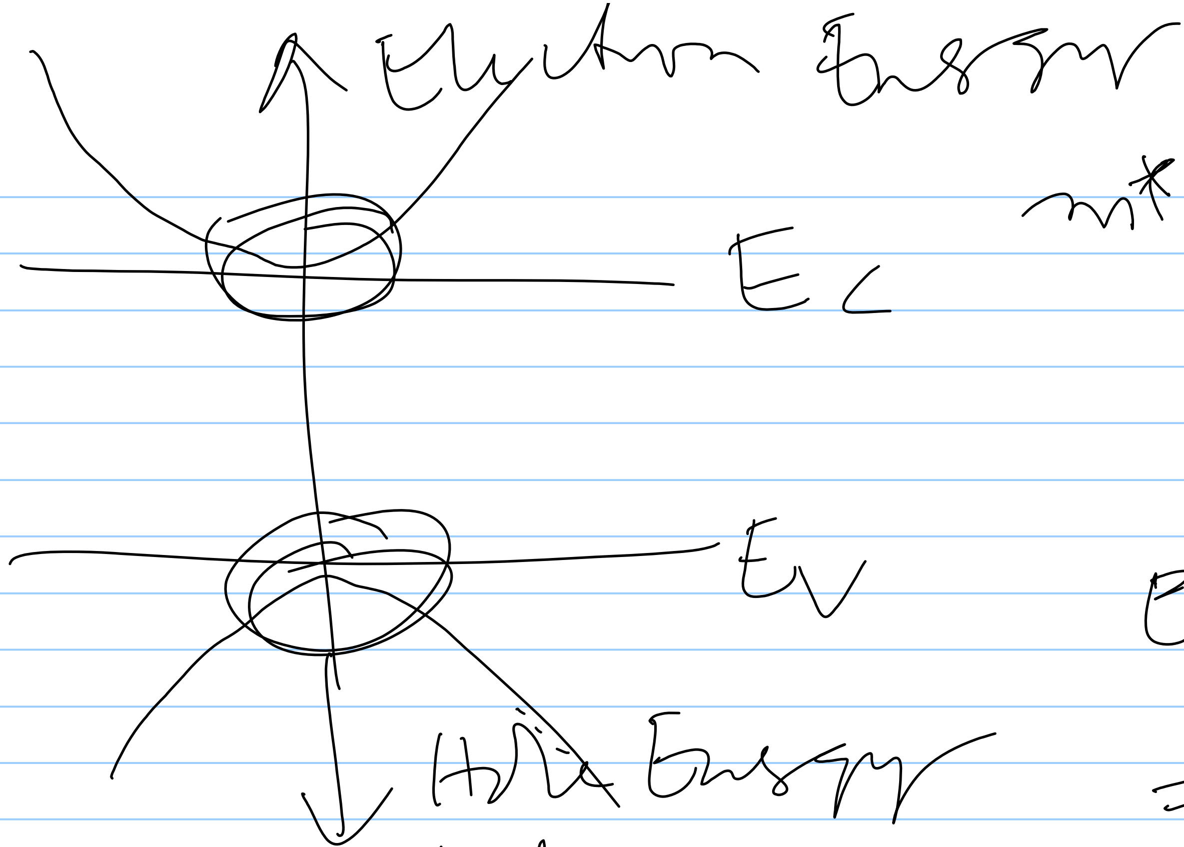
Effective mass

$$\Rightarrow \frac{dE}{dk} = \frac{\hbar^2 k}{m}$$

$$m_e^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$$

$$\Rightarrow \frac{d^2E}{dk^2} = \hbar^2/m$$

$$\Rightarrow m = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$$



$$m^* = \frac{\hbar^2}{\frac{d^2E}{dk^2}}$$

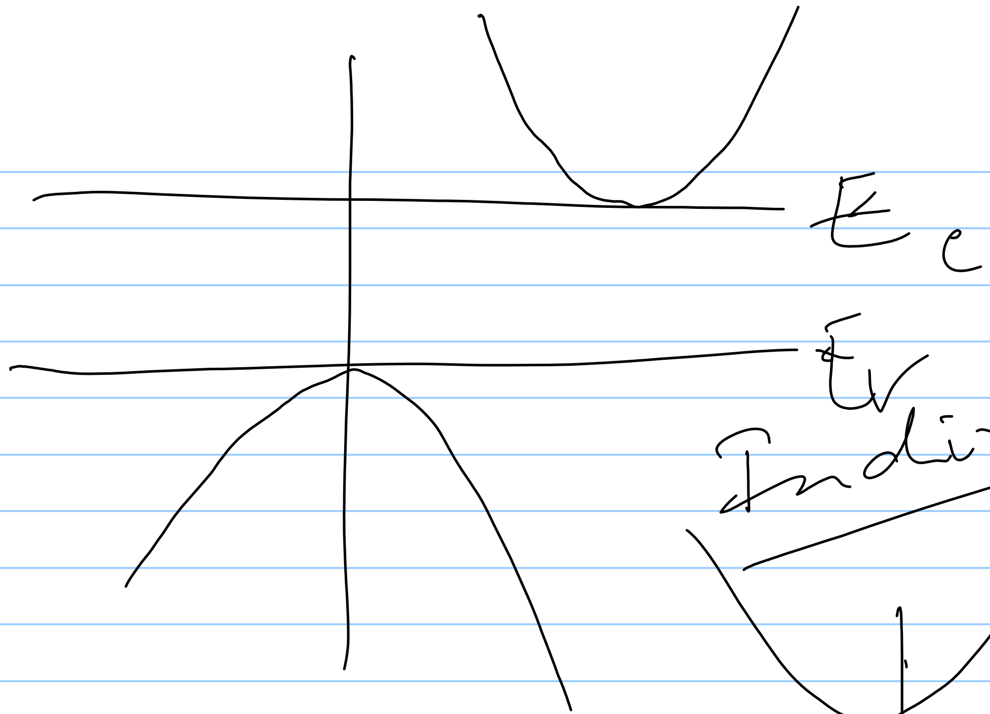
$$E = E_v - \frac{\hbar^2 k^2}{2m^*}$$

$$= E_v + \frac{\hbar^2 k^2}{2(-m^*)}$$

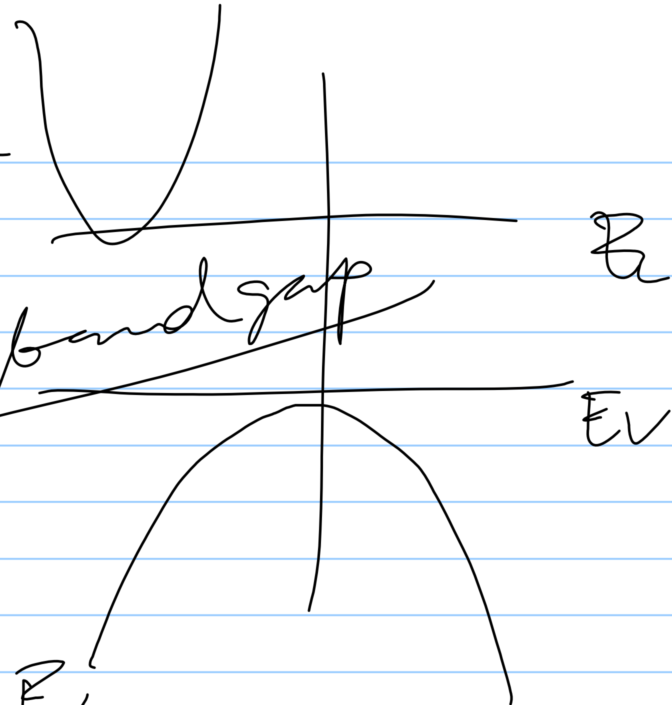
Negative electron energy is increasing

$$-E' = \frac{\hbar^2 k^2}{2m_h^*}$$

Si

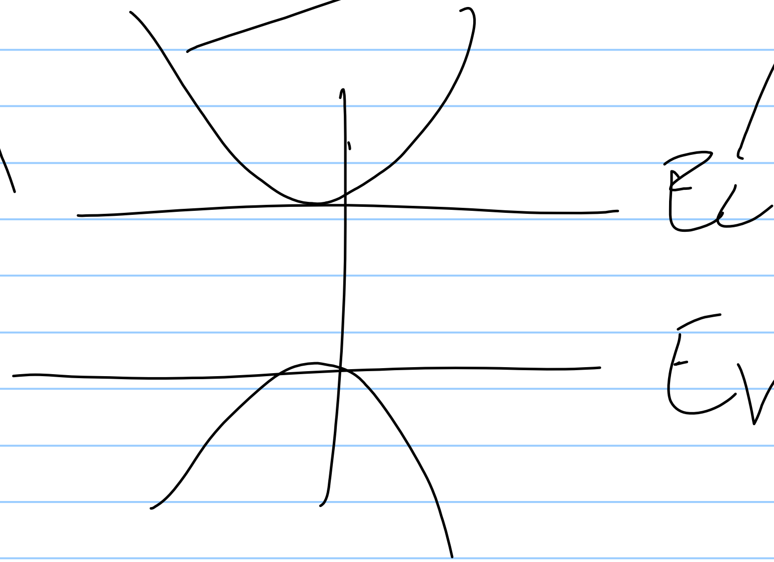


Ge

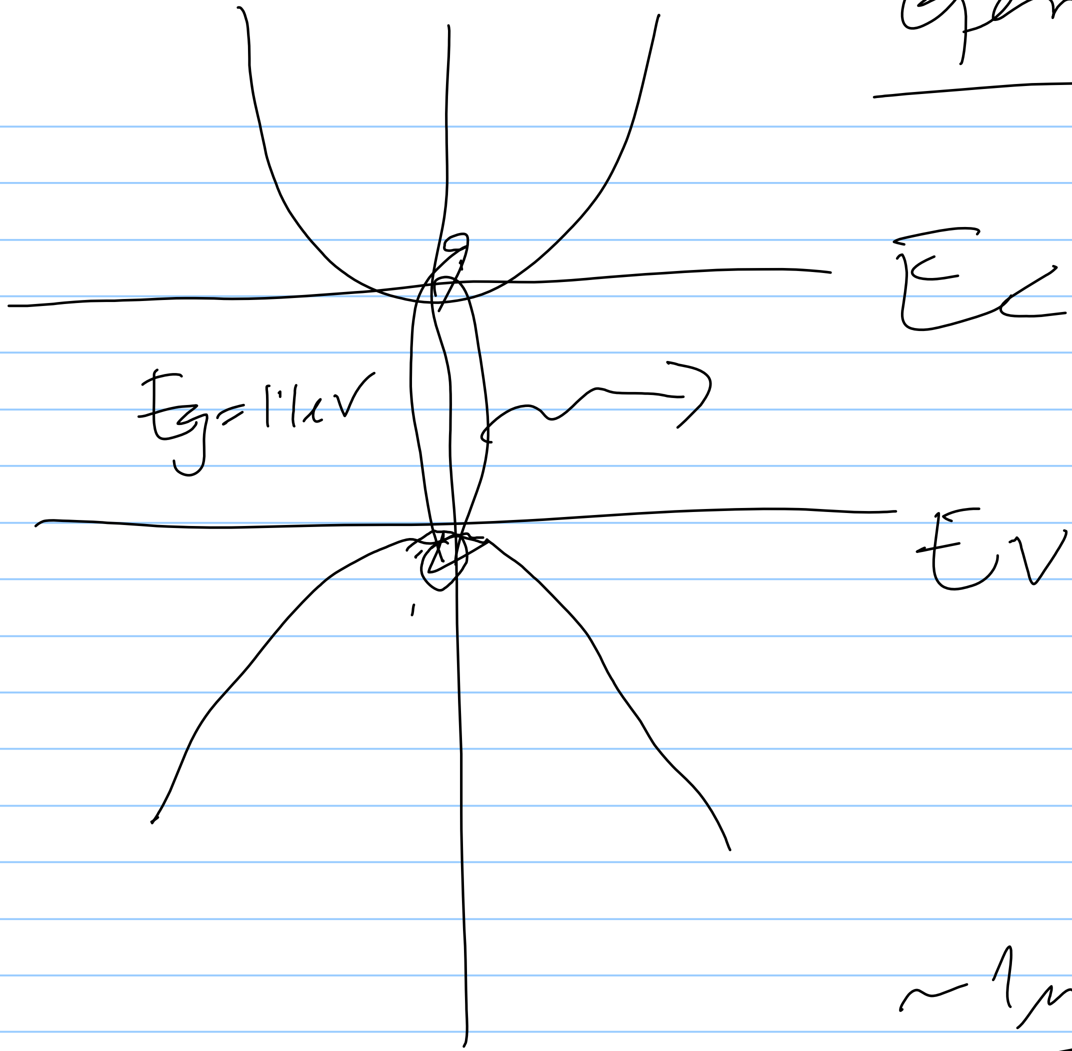


Indirect bandgap

GaAs



Direct bandgap Semi Cond.



Generation / Recombination

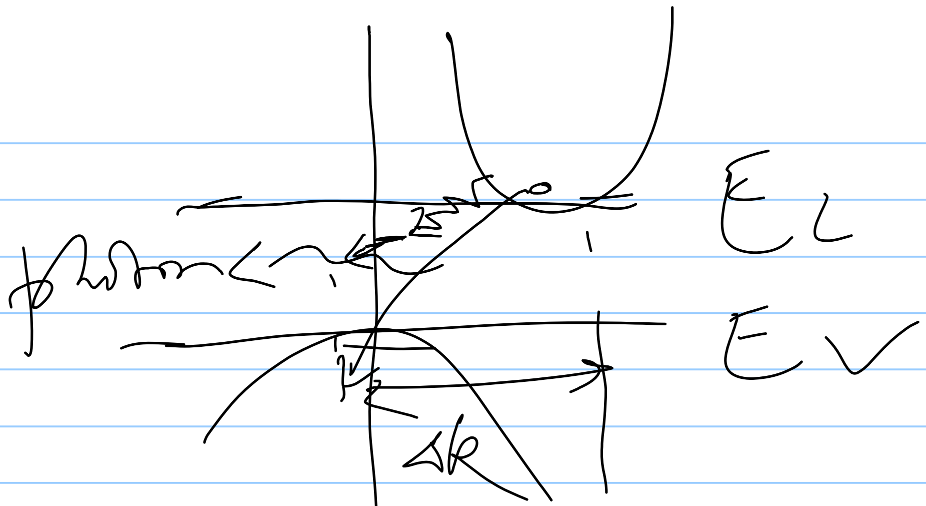
Energy Conservation

Momentum Conservation

$$\lambda = \frac{hc}{E_g}$$

$$= \frac{4.14\times10^{-15}\ eV\,s \cdot c}{1.1\ eV}$$

$$\sim 1\,\mu m = \frac{4.14\times10^{-15} \cdot 3.8\times10^{10}\ cm}{1.1}$$



$a \sim 1\,\text{Å}$

$$p_{photon} = \frac{h}{\lambda} = h/nm$$

$$\Delta p_{electron} = \hbar\,\Delta k = \frac{h}{2\pi}\,\frac{\pi}{a} = \frac{h}{2a}$$

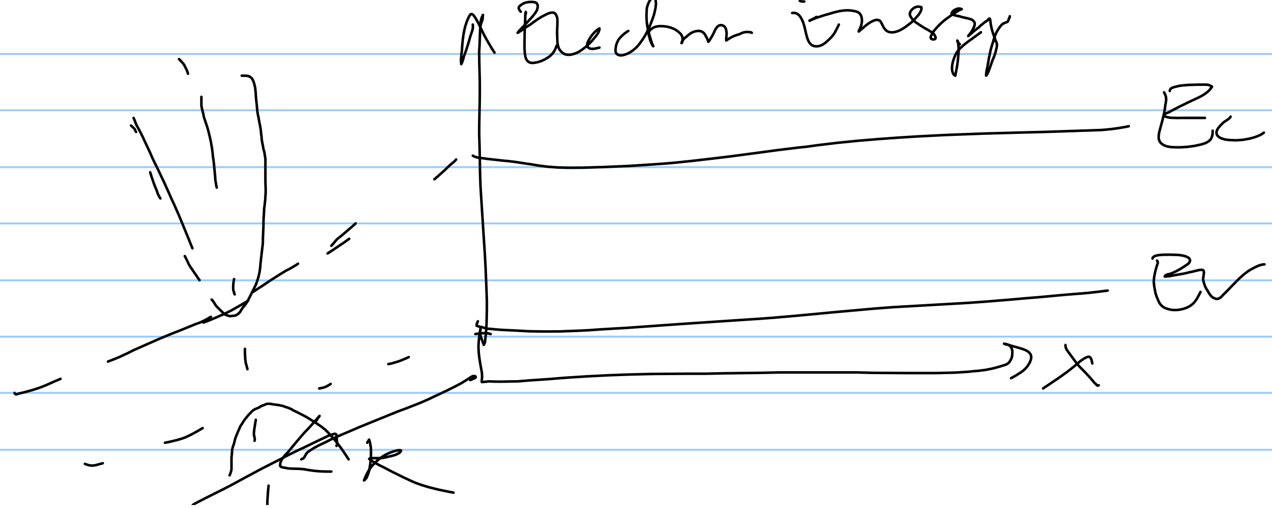
$$= 10^4\,p_{photon}$$

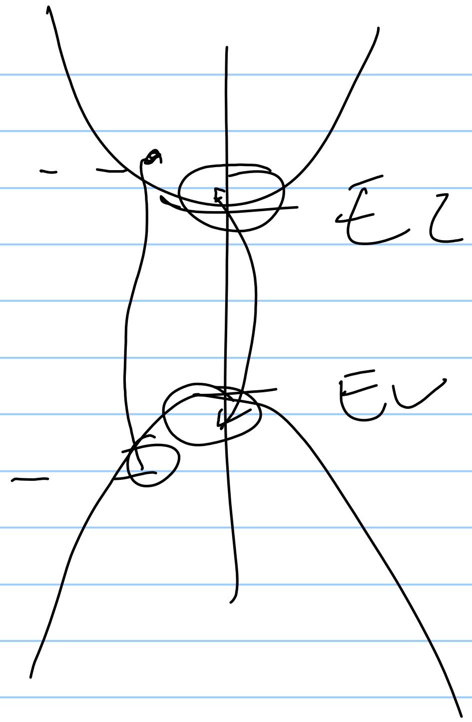
$$\Delta p_{electron} \gg p_{photon}$$

Phonon wave

packets of vibration

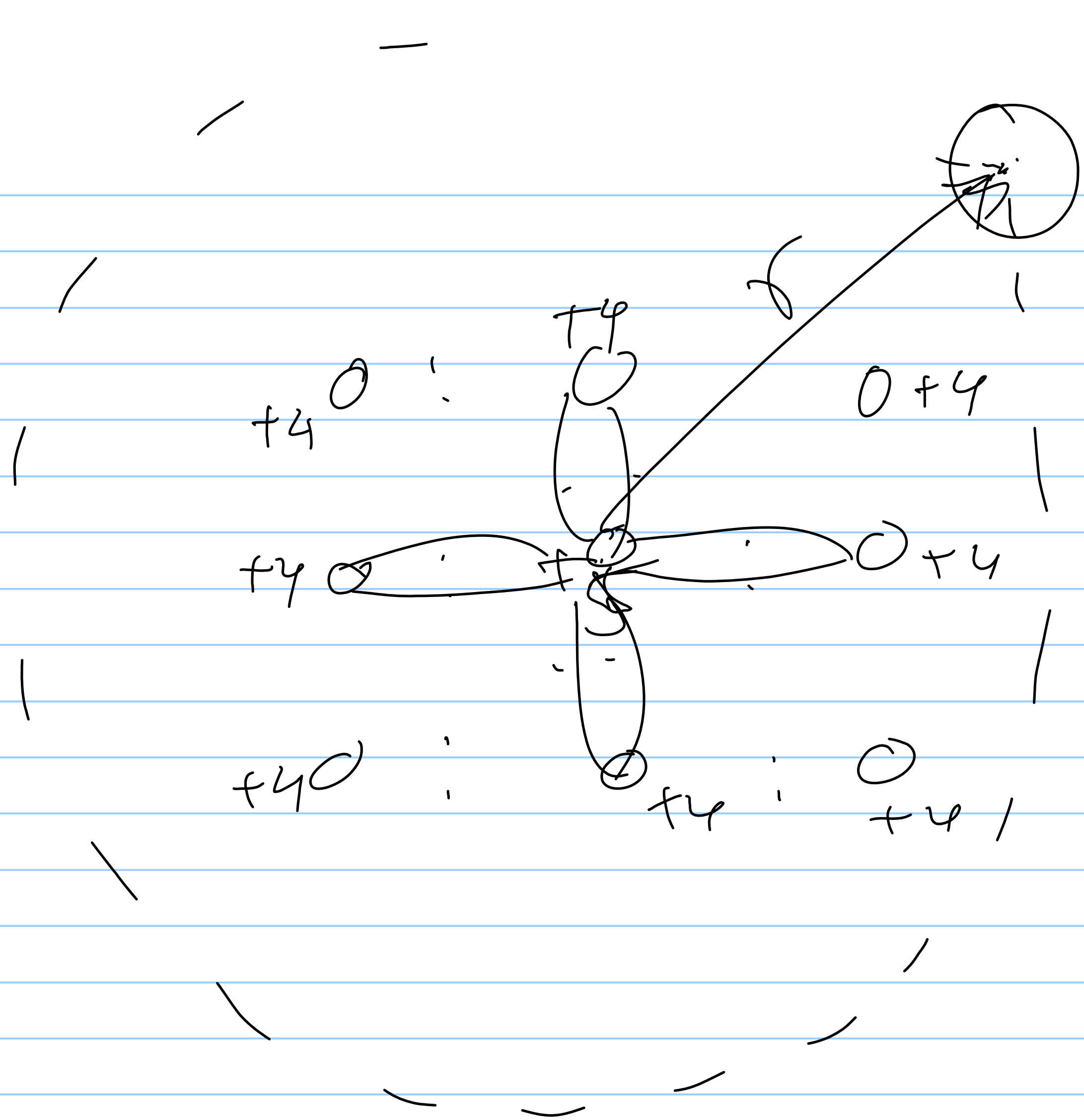
$$p_{phonon} = \frac{h}{\lambda_{phonon}} \sim a$$





Donor level

Acceptor level



$$\frac{mv^2}{r} = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

$$mvr = n\hbar$$

$$\frac{m n^2 \hbar^2}{m^2 r^2 \cdot r} = \frac{e^2}{4\pi\varepsilon_0 r^2}$$

$$r = \frac{4\pi\varepsilon_0 n^2 \hbar^2}{m e^2}$$

$$T.E = P.E + K.E$$

$$= -\frac{q^2}{4\pi\epsilon_0 r} + \frac{1}{2} m \frac{n^2\hbar^2}{m^2 r^2}$$

$$= -\frac{q^2\, m q^2}{(4\pi\epsilon_0)^2 n^2 \hbar^2} + \frac{1}{2}\frac{n^2\hbar^2 (m q^2)^2}{m\,(4\pi\epsilon_0)^2 (n^2\hbar^2)^2}$$

$$= -\frac{1}{2}\frac{m q^4}{(4\pi\epsilon_0)^2 n^2 \hbar^2}$$

$\epsilon_0 = 8.85 \times 10^{-14}\ F/cm$

$\epsilon_{Si} = 11.8\,\epsilon_0$

$E_{ion}\Big|_H = 13.6\,eV$

$m \leftarrow m_e^*$

$\epsilon_0 \leftarrow \epsilon_r \epsilon_0$

$E_{ion}\Big|_{Si} \simeq \dfrac{13.6\,eV}{(\epsilon_r)^2} \approx 0.1\,eV$

$0.046\,eV$